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The dipole-exchange spin wave spectrum for anisotropic ferromagnetic films with mixed exchange boundary conditions

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Abstract. A theory is developed for dispersion characteristics of spin waves in ferromagnetic films taking into account both dipole–dipole and exchange interactions, crystallographic anisotropy and mixed exchange boundary conditions on the film surfaces. An arbitrary orientation of the external bias magnetic field with respect to the orientation of crystallographic axes and the film normal is assumed. The influence of crystallographic anisotropy on the spin wave spectrum of a ferromagnetic film is discussed. The theoretical results obtained are compared with the results of experiments performed in yttrium–iron garnet (YIG) and Mn–ferrospinel films.

1. Introduction

Recent experiments performed in the thin monocrystalline yttrium–iron garnet (YIG) films demonstrated that the spin wave (SW) excitation and propagation in such films could be affected simultaneously by several relevant factors: dipole–dipole interaction, inhomogeneous exchange interaction, spin pinning conditions (or exchange boundary conditions) which are determined by the surface anisotropy, and the crystallographic anisotropy in the film volume (see e.g. Adam *et al* 1978, Andreev *et al* 1984, Kalinikos *et al* 1983).

In particular, the paper by Andreev *et al* 1984 showed that the width of the dipole 'gaps' in the dipole-exchange SW spectrum of the 0.5 μm thick YIG film depends significantly on the orientation of the crystallographic axes of the film with respect to the orientation of the bias magnetic field.

The existing theory of SW spectra in ferromagnetic films (FF) considers the influence of crystallographic anisotropy on the SW spectrum mostly in the non-exchange limit (see e.g. Sneider 1972, Vittoria and Wilsey 1974, Bajpai *et al* 1979, Patton 1979a, b, Lemons and Auld 1981, Beregov 1984, Gieniusz and Smoczynski 1987). Several papers where crystallographic anisotropy was taken into account in the dipole-exchange theory are restricted to certain particular cases of external bias magnetic field orientation for the

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films of cubic or uniaxial ferromagnets (see e.g. Galkin *et al* 1985, Rado and Zhang 1986, Miteva *et al* 1986, Rado and Hicken 1988). So the problem needs a closer and more general consideration.

Our present paper is aimed at such a consideration. In the paper we develop dipole-exchange theory of SW spectra in FF magnetized in an arbitrary direction with arbitrary values of crystallographic and surface anisotropy, so all the relevant factors are taken into account. The paper represents an extension of our previous calculations of the dipole-exchange SW spectrum in FF (see Kalinikos and Slavin 1986) for the case of anisotropic FF having arbitrary directions of crystallographic axes.

The structure of the paper is as follows. In section 2 we formulate the problem and obtain the infinite system of algebraic equations for the amplitudes of spin wave modes—the eigenfunctions of the boundary problem for anisotropic FF. In section 3 we obtain the exact SW dispersion equation for anisotropic FF in the form of an infinite determinant and solve this equation numerically. In section 4 the same dispersion equation is represented in the form of an infinite series. A simple, approximate SW dispersion equation for anisotropic FF is obtained in section 5 using classical perturbation theory. This is followed by a discussion of the SW spectrum of anisotropic FF in the particular cases of perpendicular and tangential magnetization in sections 6 and 7 respectively. The hybridization of SW modes in anisotropic FF is considered in section 8. In the same section we discuss the influence of crystallographic anisotropy on the width of dipole 'gaps' in the dipole-exchange SW spectrum of anisotropic FF. The conclusions are given in section 9.

2. Formulation of the problem and the principal equations

We consider a ferromagnetic film of a thickness L in the ξ direction. In the other two directions (η and ζ) the film is considered to be infinite (see figure 1). The axis ζ is assumed to be parallel to the direction of propagation of the spin wave:

$$\boldsymbol{m}(\xi, \zeta, t) = \boldsymbol{m}(\xi, k_\zeta) \exp[\mathrm{i}(\omega t - k_\zeta \zeta)]. \tag{1}$$

The film is magnetized to saturation by a bias external magnetic field $\boldsymbol{H}_0$ of an arbitrary direction which is determined by the angles θ_H and φ_H. For the convenience of further analysis we introduce yet another coordinate system, x, y, z, in which the axis z is parallel to the direction of the saturation magnetization $\boldsymbol{M}_0$. The transition from the coordinate system ξ, η, ζ to the coordinate system x, y, z can be done by means of orthogonal transformations of rotation through angles φ and $(\pi/2 - \theta)$.

The matrices of these transformations are of the form

$$\mathbf{q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix} \qquad \mathbf{q} = \begin{pmatrix} \sin\theta & 0 & -\cos\theta \\ 0 & 1 & 0 \\ \cos\theta & 0 & \sin\theta \end{pmatrix}.$$

Here and henceforth tensors and matrices are denoted as bold, sans serif characters. The angles φ and θ characterizing the equilibrium orientation of the saturation magnetization $\boldsymbol{M}_0$ are determined from the minimization procedure for the density of magnetic energy of the anisotropic FF (see the appendix).

We shall derive expressions for the spectrum $\omega = f(k_\zeta)$ of the non-uniform plane SW (1) in FF taking into account dipole–dipole interaction and inhomogeneous exchange

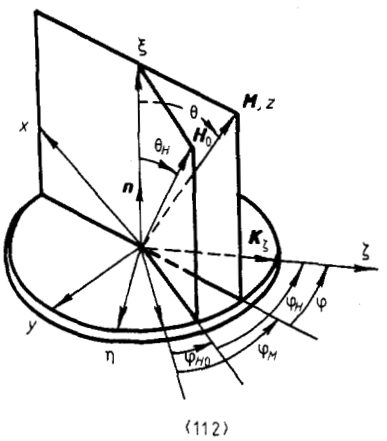

Figure 1. The geometry and orientation of the coordinate axes.

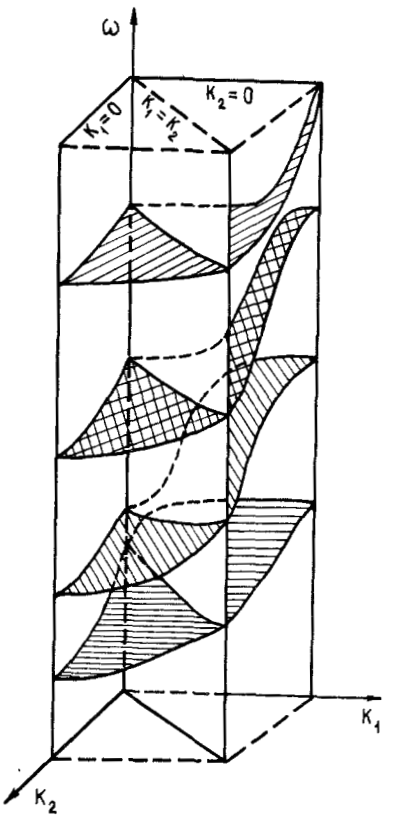

Figure 2. Transformation of the dipole-exchange SW spectrum in tangentially magnetized ferromagnetic film with the change in the direction of propagation of the spin wave with respect to the direction of equilibrium magnetization $\boldsymbol{M}_0$ ($\boldsymbol{k}_1 \perp \boldsymbol{M}_0$, $\boldsymbol{k}_2 \| \boldsymbol{M}_0$).

interaction as well as the crystallographic anisotropy in the film volume and exchange boundary conditions on the film surfaces.

The total effective magnetic field in the Landau–Lifshitz equation of motion for magnetization (see e.g. Lax and Batton 1962, Kalinikos 1981) in our case has the form:

$$\boldsymbol{H}^{\text{eff}}(\boldsymbol{r}, t) = \boldsymbol{H}_i + \boldsymbol{h}_d(\boldsymbol{r}, t) + \boldsymbol{h}_{ex}(\boldsymbol{r}, t) + \boldsymbol{h}^a(\boldsymbol{r}, t) \tag{2}$$

where $\boldsymbol{H}_i = \boldsymbol{H}_0 + \boldsymbol{H}_{d0} + \boldsymbol{H}_0^a$ is a constant internal magnetic field consisting of an external bias magnetic field $\boldsymbol{H}_0$, a constant dipole (or demagnetization) field $\boldsymbol{H}_{d0}$, and a constant field of the crystallographic anisotropy $\boldsymbol{H}_0^a$. The direction of the constant internal magnetic field $\boldsymbol{H}_i$ coincides with the direction of the constant magnetization $\boldsymbol{M}_0$. All the other components of the effective magnetic field (2) are variables: $\boldsymbol{h}_d(\boldsymbol{r}, t)$ is a variable dipole field, $\boldsymbol{h}_{ex} = \alpha\nabla^2\boldsymbol{m}(\boldsymbol{r}, t)$ is a variable exchange field (where α is the exchange constant and $\boldsymbol{m}(\boldsymbol{r}, t)$ is the variable magnetization in the FF), and $\boldsymbol{h}^a$ is a variable field of crystallographic anisotropy.

The effective fields of crystallographic anisotropy can be calculated if the expressions for the densities of energy of anisotropy E_A and E_u are known. For the linear problems

when the condition $\boldsymbol{m}(\boldsymbol{r}, t) \ll \boldsymbol{M}_0$ is observed there is, however, an easier way to calculate the effective magnetic fields created by crystallographic anisotropy—namely the method of effective demagnetization factors (see e.g. Gurevich 1973). In the framework of this method the effective magnetic field of anisotropy $\boldsymbol{H}^{\mathrm{a}}(\boldsymbol{r}, t)$ is represented in the form

$$\boldsymbol{H}^{\mathrm{a}}(\boldsymbol{r}, t) = \boldsymbol{H}_0^{\mathrm{a}} + \boldsymbol{h}^{\mathrm{a}}(\boldsymbol{r}, t) = -\mathbf{N}^{\mathrm{a}}\boldsymbol{M}_0 - \mathbf{N}^{\mathrm{a}}\boldsymbol{m}(\boldsymbol{r}, t) \tag{3}$$

where $\mathbf{N}^{\mathrm{a}}$ is the tensor of effective demagnetization factors of crystallographic anisotropy. The components of the tensor $\mathbf{N}^{\mathrm{a}}$ can be demagnetization factors of any type of anisotropy or the sum of demagnetization factors of different types of anisotropy taken into account simultaneously.

For example, in the case of a cubic monocrystalline film having induced uniaxial anisotropy, the components of the tensor $\mathbf{N}^{\mathrm{a}}$ can be represented as a sum of demagnetization factors of cubic anisotropy N^{A} and uniaxial anisotropy N^{u}

$$N^{\mathrm{a}} = N^{\mathrm{A}} + N^{\mathrm{u}}.$$

Determining components of the tensor $\mathbf{N}^{\mathrm{a}}$ for each type of anisotropy may be considered in this context as a separate problem. The solution of such a problem for the case of a ferromagnetic ellipsoid having cubic and uniaxial anisotropy and the arbitrary direction of the crystallographic axes with respect to its geometrical axes is presented in Belyakov (1984, 1986). We shall use this solution here to determine the effective magnetic fields for different types of crystallographic anisotropy in FF.

In accordance with Kalinikos and Slavin (1986) we shall rewrite the initial system of equations consisting of the linearized Landau–Lifshitz equation of motion for magnetization Maxwell equations in the magnetostatic limit and the usual electrodynamic boundary conditions in the form of a matrix integral–differential equation for vector Fourier amplitude of the spin wave (1):

$$\mathbf{F}\boldsymbol{m}(\xi, k_\zeta) = \boldsymbol{h}_{\mathrm{d}}(\xi, k_\zeta), \tag{4}$$

where $\boldsymbol{h}_{\mathrm{d}}(\xi, k_\zeta)$ is a Fourier amplitude of the variable dipole magnetic field which is determined by the expression (see Kalinikos and Slavin 1986)

$$\boldsymbol{h}_{\mathrm{d}}(\xi, k_\zeta) = \int_{-L/2}^{L/2} \mathbf{G}(\xi, \xi', k_\zeta)\boldsymbol{m}(\xi', k_\zeta)\,\mathrm{d}\xi'. \tag{5}$$

The tensorial Green function of the Maxwell equations in the magnetostatic limit $\mathbf{G}$ is defined in Kalinikos (1981).

After separating the diagonal part $\mathbf{J}$ of the linear matrix-differential operator $\mathbf{F}$ in the coordinate system x, y, z we may represent equation (4) in the following form:

$$\mathbf{J}\boldsymbol{m}(\xi) = -\mathbf{T}^{\mathrm{a}}\boldsymbol{m}(\xi) + \int_{-L/2}^{L/2} \mathbf{G}_{xy}(\xi, \xi')\boldsymbol{m}(\xi')\,\mathrm{d}\xi' \tag{6}$$

where

$$\mathbf{J} = (-\alpha\partial^2/\partial\xi^2 + \alpha k^2 + \omega_H/\omega_M)\mathbf{I}$$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \mathbf{T}^{\mathrm{a}} = \begin{pmatrix} N_{xx}^{\mathrm{a}} & (-\mathrm{i}\omega/\omega_M + N_{xy}^{\mathrm{a}}) \\ (\mathrm{i}\omega/\omega_M + N_{yx}^{\mathrm{a}}) & N_{yy}^{\mathrm{a}} \end{pmatrix}$$

$$\omega_H = \gamma H_{\mathrm{i}} \qquad \omega_M = \gamma M_0 \qquad \gamma = |g|\mu_0 \qquad H_{\mathrm{i}} = H_{0z} + H_{\mathrm{d}0z} - N_{zz}^{\mathrm{a}} M_0.$$

H_{0z} is the z-projection of the external bias magnetic field H_0, $H_{\mathrm{d}0z}$ is the z projection of

the constant demagnetization field H_{d0}, $|g|$ is the modulus of the gyromagnetic ratio for electron spin and μ_0 is the permeability of vacuum.

To derive the expressions for the dipole-exchange SW spectrum of a ferromagnetic film we may integrate equation (6) along with the exchange boundary conditions which depend on the surface anisotropy. In the case of uniaxial surface anisotropy the exchange boundary conditions are given by (see Soohoo 1965):

$$\begin{aligned} &\partial m^x(\xi)/\partial\xi + d\cos^2\theta m^x(\xi) = 0 \\ &\partial m^y(\xi)/\partial\xi + d\cos^2\theta m^y(\xi) = 0 \end{aligned} \tag{7}$$

where d is a parameter of spin pinning on the film surface.

In accordance with Kalinikos and Slavin 1986 we shall solve (6) by expanding $\boldsymbol{m}(\xi)$ in the infinite series of complete orthogonal vector functions—spin wave modes (SWM) $\boldsymbol{S}_n^p(\xi)$:

$$\boldsymbol{m}(\xi) = M_0 \sum_{n,p} m_n^p \boldsymbol{S}_n^p(\xi) \qquad p = x, y. \tag{8}$$

SWM are the eigenfunctions of the diagonal matrix-differential operator $\mathbf{J}$ and they satisfy the exchange boundary conditions (7) on both film surfaces (see Kalinikos and Slavin 1986).

Substituting (8) for the $\boldsymbol{m}(\xi)$ in equation (6) and using the orthogonality of the SWM we obtain the infinite system of algebraic equations for the SWM vector amplitudes $\boldsymbol{m}_n = (m_n^x, m_n^y)$:

$$\mathbf{D}_{nn}\boldsymbol{m}_n + \sum_{n\neq n'} \mathbf{R}_{nn'}\boldsymbol{m}_{n'} = 0 \qquad n, n' = 1, 2, 3 \ldots \tag{9}$$

where

$$\mathbf{D}_{nn} = \begin{pmatrix} (J_n^x + \sin^2\theta + AP_{nn}^{xx} + N_{xx}^a) & [(CP_{nn}^{xy} + N_{xy}^a T_{nn}^{xy}) \\ & \quad - i(T_{nn}^{xy}\omega/\omega_M - DQ_{nn}^{xy})] \\ [(CP_{nn}^{yx} + N_{yx}^a T_{nn}^{yx}) & (J_n^y + EP_{nn}^{yy} + N_{yy}^a) \\ \quad + i(T_{nn}^{yx}\omega/\omega_M + DQ_{nn}^{yx})] & \end{pmatrix} \tag{10}$$

$$\mathbf{R}_{nn'} = \begin{pmatrix} (AP_{nn'}^{xx} + iBQ_{nn'}^{xx}) & [(CP_{nn'}^{xy} + N_{xy}^a T_{nn'}^{xy}) \\ & \quad - i(T_{nn'}^{xy}\omega/\omega_M - DQ_{nn'}^{xy})] \\ [(CP_{nn'}^{yx} + N_{yx}^a T_{nn'}^{yx}) & (EP_{nn'}^{yy}) \\ \quad + i(T_{nn'}^{yx}\omega/\omega_M + DQ_{nn'}^{yx})] & \end{pmatrix}. \tag{11}$$

$J_n^p = \Omega_n^p/\omega_M$ are the eigenvalues of the boundary problem (7) for the differential operator $\mathbf{J}$; $n, n' = 1, 2, 3 \ldots$; $p, p' = x, y$. The frequency Ω_n^p is given by the equation

$$\Omega_n^p = \omega_H + \alpha\omega_M(k_n^p)^2 \tag{12}$$

where $(k_n^p)^2 = k^2 + (\kappa_n^p)^2$, and the transverse component κ_n^p of the full SW wavevector $\boldsymbol{k}_n^p$ is determined by equation (15) in Kalinikos and Slavin (1986).

The expressions for matrix elements $P_{nn'}^{pp'}$, $Q_{nn'}^{pp'}$, $T_{nn'}^{pp'}$ and for the angle functions A, B, C, D, E are also given in Kalinikos and Slavin (1986). The effective demagnetization factors of anisotropy $N_{pp'}^a$ are presented in Belyakov (1984, 1986) and will be written below for all the particular cases under consideration.

The infinite system (9) gives the exact description of the linear SW processes in anisotropic FF and enables us to obtain the expressions for the SW spectrum and for the transverse (along the axis ξ) distribution of the variable magnetization (8) in a ferromagnetic film. In the isotropic limit ($N^{a}_{pp'} = 0$) this system coincides with the infinite system obtained in Kalinikos and Slavin (1986) (see equation (22) in Kalinikos and Slavin (1986)).

3. SW dispersion equation in the form of an infinite determinant

The condition of the vanishing of the determinant of the infinite system (9) yields the exact dispersion equation for propagating dipole-exchange SW in anisotropic FF. So the problem of determination of the exact dispersion law for SW in anisotropic FF is reduced to the problem of calculating eigenvalues of the block matrix of the infinite system (9). The calculation of eigenvalues of such a matrix could be done by means of the well known QR algorithm (see e.g. Wilkinson and Reinsch 1976).

The numerical analysis of the system (9) shows that the SW spectrum of anisotropic FF in the coordinates (ω, $\boldsymbol{k}$) (where ω is a SW frequency and $\boldsymbol{k}(k_1, k_2)$ is a two-dimensional SW wavevector lying in the film plane) consists of the non-crossing surfaces $\omega_n(k_1, k_2)$ with centres corresponding to the frequencies of the spin wave resonance modes $\omega_n(k_1 = k_2 = 0)$. These dispersion surfaces can be numbered using the numbers of corresponding modes of spin wave resonance. The qualitative example of the SW spectrum of a tangentially magnetized FF is presented in figure 2. While moving along the dispersion surface, either along the radius ($\boldsymbol{k}/|\boldsymbol{k}|$ = const.) or along the azimuth ($|\boldsymbol{k}|$ = const.) we shall find that SW dispersion and transverse distribution of the variable magnetization change considerably. We stress that the curvature of dispersion surface and the form of dispersion curve corresponding to the given direction of SW propagation are determined by competition between dipole–dipole interaction, inhomogeneous exchange interaction and influence of crystallographic anisotropy. Thus it is not possible to draw a general system of dispersion surfaces for dipole-exchange SW in anisotropic FF of arbitrary thickness and of arbitrary direction of $\boldsymbol{M}_0$.

The numerical calculations of the SW spectrum for the tangentially magnetized† anisotropic FF with unpinned surface spins ($d_1 = d_2 = 0$) were done using the QR algorithm (see Wilkinson and Reinsch 1976). The accuracy of the numerical calculations was determined by the order of reduction of the system (9) (i.e. by the dimension of the finite determinant which was used in the calculations instead of the full infinite determinant of the system (9)) and was not worse than 0.01%.

An example of calculated SW dispersion curves for the transverse SW ($\varphi = 90°$, $\theta = 90°$) in a thin ($L = 0.5$ μm) YIG film having uniaxial anisotropy is presented in figure 3. In this particular case the effective demagnetization factors of anisotropy have the values (see Belyakov 1986 and (A6) in the appendix of this paper)

$$N^{a}_{xx} = 2H^{u}/M_0 \qquad N^{a}_{yy} = N^{a}_{zz} = N^{a}_{xy} = 0.$$

As in the case of isotropic FF the dipole-exchange SW spectrum of anisotropic FF is discrete due to the exchange interaction. The uniaxial anisotropy modifies the dipole-exchange SW spectrum so that some of the dispersion curves have regions of 'negative'

† We assume that in anisotropic FF 'tangential' magnetization means that the equilibrium magnetization $\boldsymbol{M}_0$ is oriented tangentially to the film surface.

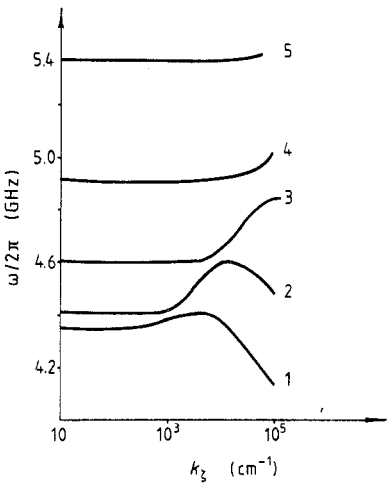

Figure 3. Spectrum of transverse dipole-exchange SW in tangentially magnetized YIG film having uniaxial perpendicular anisotropy and unpinned surface spins. $H_0 = 1000$ Oe; $4\pi M_0 = 1750$ Oe; $H^u = 170$ Oe; $L = 0.5\ \mu$m.

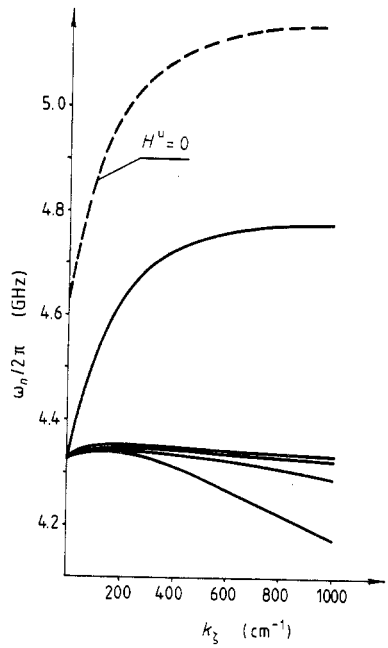

Figure 4. Spectrum of non-exchange transverse SW in tangentially magnetized YIG film having uniaxial anisotropy. The dispersion curve having positive group velocity corresponds to the surface SW; the set of curves having negative group velocity corresponds to the volume SW (see Beregov 1984). $H_0 = 1000$ Oe; $4\pi M_0 = 1750$ Oe; $H^u = 170$ Oe; $L = 20\ \mu$m.

dispersion. In the particular case presented in figure 3 the dispersion branches numbered 1, 2, 3 have regions of negative dispersion ($\partial\omega/\partial k < 0$).

The exchange branch splitting may be neglected for the sufficiently thick FF so that in the longwave part of the spectrum the SW dispersion for such films could be considered in the non-exchange ($\alpha = 0$) limit. Figure 4 shows the spectrum of non-exchange transverse SW in YIG film of the thickness $L = 20\ \mu$m having uniaxial anisotropy. In this case the SW spectrum consists of the infinite set of volume SW having 'negative' dispersion and only one quasi-surface SW having 'positive' dispersion. This result is in good agreement with the results of the existing non-exchange theory (see Beregov 1984, Beregov and Kudinov 1987, Gieniusz and Smoczynski 1987).

The influence of cubic anisotropy on the spectrum of transverse SW in FF of the same thickness is illustrated by figure 5. The calculations were made for the YIG film of orientation (111) (which means that the axis ⟨111⟩ is perpendicular to the film plane) when the equilibrium magnetization was directed along the axis ⟨110⟩. The components of the tensors $\mathbf{N}^a$ for this case have the values (see Belyakov 1984 and (A6) in appendix)

$$N^a_{xx} = -2H^A/M_0 \qquad N^a_{yy} = N^a_{zz} = -H^A/M_0 \qquad N^a_{xy} = \sqrt{2}H^A/M_0.$$

As is shown in figure 5, cubic anisotropy leads to the appearance of propagating volume

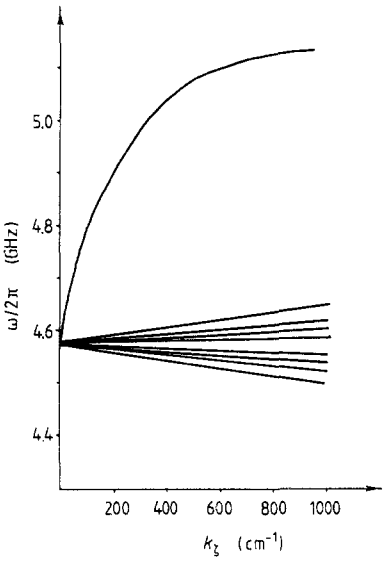

Figure 5. Spectrum of non-exchange transverse SW in tangentially magnetized YIG film of orientation (111) having cubic anisotropy ($\boldsymbol{M}_0$ is parallel to the axis ⟨110⟩). $H_0 = 1000$ Oe; $4\pi M_0 = 1750$ Oe; $H^A = -42$ Oe ($\boldsymbol{M}_0$ is parallel to the axis ⟨110⟩). $H_0 = 1000$ Oe; $4\pi M_0 = 1750$ Oe; $H^A = -42$ Oe; $L = 20\ \mu$m.

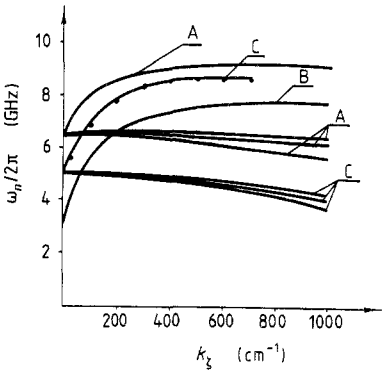

Figure 6. Spectrum of non-exchange transverse SW in Mn–ferrospinel film of orientation (100). A: $\boldsymbol{M}_0$ parallel to the axis ⟨110⟩; B: $\boldsymbol{M}_0$ parallel to the axis ⟨100⟩; C: $-\varphi_M = 25°$; full circles: experimental data from Anfinogenov *et al* (1986). $H_0 = 600$ Oe; $4\pi M_0 = 5200$ Oe; $H^A = -200$ Oe; $L = 29\ \mu$m.

SW modes having both 'positive' and 'negative' dispersion. Similar results were obtained in Beregov and Kudinov 1987, Gieniusz and Smoczynski 1987.

Let us consider now a case of a tangentially magnetized FF of orientation (100). There is experimenal data for such a case in Anfinogenov *et al* (1986) for a 29 μm thick Mn–ferrospinel film. It is interesting to compare these experimental results with the results of our calculations. The components of the tensor $\mathbf{N}^a$ for this case are (Belyakov 1984)

$$N^a_{xx} = N^a_{xy} = 0 \qquad N^a_{yy} = 3(\cos 4\varphi_M - 1)H^A/2M_0$$

$$N^a_{zz} = -(\cos 4\varphi_M + 3)H^A/2M_0$$

where φ_M is the angle between the $\boldsymbol{M}_0$ and the axis ⟨100⟩.

The results of our calculations are shown in figure 6 by full curves. Full circles in figure 6 show the experimental results of Anfinogenov *et al* (1986). Full curves marked by letters A, B, C correspond to the different orientations of $\boldsymbol{M}_0$ in the film plane (i.e. to the different values of φ_M). The calculations show that the minimum value of SW frequency is achieved when $\boldsymbol{M}_0$ is parallel to the axis ⟨100⟩. In this case the SW spectrum consists of only one SW mode of quasi-surface-type (curve B in figure 6). For all the other orientations of $\boldsymbol{M}_0$ the SW spectrum consists of one quasi-surface SW mode and an infinite number of volume SW modes (curves A, C in figure 6). The best coincidence between the experimental results and the results of our calculations for the quasi-surface SW is observed when $\varphi_M = 25°$ (curve C in figure 6). This enables us to determine the actual orientation of the equilibrium magnetization $\boldsymbol{M}_0$ with respect to the crystallographic axis in the experimental sample. The volume SW modes predicted by the theory were not

observed in the experiment of Anfinogenov *et al* (1986), probably due to the very low values of group velocity for these modes. The latest experimental results for ferrospinel films by Anfinogenov *et al* 1988 also appear to be in good agreement with the results of the above calculations. As it follows from analysis of the equation (9) for the case $\theta = 90°$, $\varphi = 90°$ the volume SW modes propagating perpendicularly to the vector $\boldsymbol{M}_0$ in the anisotropic film appear only in the case when one (or both) of the following conditions are satisfied

$$N^a_{xx} - N^a_{yy} \neq 0 \qquad N^a_{xy} \neq 0.$$

In the case when $N^a_{xx} - N^a_{yy} \neq 0$ the sign of SW dispersion coincides with the sign of this difference. For example, the SW modes in figures 4 and 6 having 'negative' dispersion correspond to the negative value of the difference $N^a_{xx} - N^a_{yy}$.

4. SW dispersion equation in the form of an infinite series

The infinite system of equations (9) enables us to obtain the exact dispersion equation for SW in anisotropic FF not only in the form of an infinite determinant but also in the form of an infinite series. The method of transformation of an infinite determinant into an infinite series is described in Vendik and Chartorizhskii (1970). Let us write as an example a dispersion equation in the form of an infinite series for the case of longitudinal ($\varphi = 0$) SW propagating in FF with totally unpinned ($d_1 = d_2 = 0$) or totally pinned ($d_1 = d_2 = \infty$) surface spins:

$$\sum_n \frac{\omega_M(\Omega_n + \omega_M N^a_{yy}) l_{nk} \sin 2\theta}{(\Omega_n + \omega_M N^a_{yy})(\Omega_n + \omega_M N^a_{xx} + \omega_M k^2_\xi \sin^2\theta/k^2_n + \omega_M \sin^2\theta) - \omega^2_M (N^a_{xy})^2 - \omega^2_n} = -1 \tag{13}$$

where l_{nk} has different values for different pinning conditions.

(i) For the case of totally unpinned surface spins:

$$l_{nk} = k^4_\xi F_n / k^4_n (1 + \delta_{0n}) \qquad n = 0, 1, 2 \ldots ;$$

(ii) for the case of totally pinned surface spins:

$$l_{nk} = k^2_\xi \kappa^2_n F_n / k^4_n \qquad n = 1, 2, 3 \ldots$$

where

$$F_n = 2[1 - (-1)^n \exp(-k_\xi L)]/k_\xi L$$

and δ_{0n} is a Kronecker delta.

The series in equation (13) converges and may be used in the numerical calculations as it stands. It is also possible to reduce (13) to the transcendental dispersion equation although the transcendental form of the equation is not convenient for numerical calculations.

5. Approximate SW dispersion equation in anisotropic FF

In accordance with Kalinikos and Slavin (1986) we can use classical perturbation theory for the approximate solution of the system (9). In the first approximation of the perturbation theory where frequency degeneracy of different SW modes is not taken into

account, we obtain the explicit approximate dispersion equation $\omega_n = f(k_\zeta, \kappa_n)$ for dipole-exchange SW in anisotropic FF with mixed exchange boundary conditions from the condition $\det \mathbf{D}_n = 0$ in the form

$$(T^{yx}_{nn}\omega_n/\omega_M + DQ^{yx}_{nn})^2 = (J^x_n + \sin^2\theta + AP^{xx}_{nn} + N^a_{xx})(J^y_n + EP^{yy}_{nn} + N^a_{yy}) - (CP^{yx}_{nn} + N^a_{yx}T^{yx}_{nn})^2. \tag{14}$$

In the case when both components of the magnetization vector $\boldsymbol{m}$ (m^x and m^y) are pinned uniformly on each side of FF the dispersion equation (14) can be rewritten in the form of a well known dispersion equation for the SW in the unlimited ferromagnetic media (see Herring and Kittel 1951):

$$\omega_n^2 = A_n^2 - |B_n|^2 = \Omega_n[\Omega_n + \omega_M(F_{nn} + F^a_{nn})] \tag{15}$$

where

$$A_n = \Omega_n + \omega_M(\sin^2\theta + N^a_{xx} + N^a_{yy})/2 + \omega_M P_{nn}[1 - \sin^2\theta(1 + \cos^2\varphi)]/2. \tag{16a}$$

$$B_n = A_n - \Omega_n - \omega_M(P_{nn}\sin^2\varphi + N^a_{yy}) + i\omega_M(P_{nn}\cos\theta\sin 2\varphi - N^a_{xy})/2 \tag{16b}$$

$$F_{nn} = \sin^2\theta - P_{nn}\sin^2\theta\cos^2\varphi + P_{nn}[\cos^2\theta + \omega_M(1 - P_{nn})\sin^2\varphi\sin^2\theta/\Omega_n] \tag{17a}$$

$$F^a_{nn} = N^a_{xx} + N^a_{yy} + [N^a_{xx}N^a_{yy} + N^a_{yy}\sin^2\theta - (N^a_{xy})^2]\omega_M/\Omega_n + \{N^a_{yy}[\cos^2\varphi - \sin^2\theta(1 + \cos^2\varphi)] + N^a_{xx}\sin^2\varphi - N^a_{xy}\cos\theta\sin 2\varphi\}P_{nn}\omega_M/\Omega_n. \tag{17b}$$

In the isotropic limit $F^a_{nn} = 0$, and equation (15) reduces to the approximate dispersion equation for SW in isotropic FF (see equation (45) in Kalinikos and Slavin 1986).

The first-order approximation in the perturbation theory where frequency degeneracy ($\det \mathbf{D}_{nn} = \det \mathbf{D}_{n'n'}$) of SW modes with different numbers ($n \neq n'$) is taken into account yields the secular dispersion equation of the form

$$\det \begin{pmatrix} \mathbf{D}_{nn}, & \mathbf{R}_{nn'} \\ \mathbf{R}_{n'n}, & \mathbf{D}_{n'n'} \end{pmatrix} = 0. \tag{18a}$$

This equation can also be written as follows:

$$\det \mathbf{D}_{nn} \det \mathbf{D}_{n'n'} = f(D^{pp'}_{nn'}, R^{pp'}_{nn'}) \tag{18b}$$

where the left-hand part is a product of the non-degenerate dispersion equations for the spin waves with numbers n and n', and the right-hand part describes the interaction (or hybridization) of these SW modes.

Equations (14) and (18) are rather general. They describe the dipole-exchange SW spectrum in anisotropic FF with mixed exchange boundary conditions and arbitrary directions of crystallographic axes and the equilibrium magnetization. The boundaries for applications of these approximate equations in the isotropic limit are discussed extensively in Kalinikos and Slavin (1986) and Slavin and Fetisov (1988).

In practice, as a rule, it is interesting to have the approximate dispersion equations in the explicit form for the particular cases of definite orientation of crystallographic axes and bias magnetic field in the anisotropic FF. Let us consider the dipole-exchange

SW spectrum in anisotropic FF in some particular cases using the approximate dispersion equation (14).

6. Perpendicular magnetization

In the case of perpendicular magnetization ($\theta = 0$, $J_n^x = J_n^y = J_n = \Omega_n/\omega_M$) the dispersion equation (14) may be written in the Kittel form (15), where $F_{nn} = P_{nn}$ and

$$F_{nn}^a = N_{xx}^a + N_{yy}^a + [N_{xx}^a N_{yy}^a - (N_{xy}^a)^2]\omega_M/\Omega_n + (N_{yy}^a \cos^2\varphi + N_{xx}^a \sin^2\varphi - N_{xy}^a \sin 2\varphi)P_{nn}\omega_M/\Omega_n. \tag{19}$$

As is clear from equations (15) and (19), anisotropy in perpendicularly magnetized FF can lead to the dependence of the SW eigenfrequency ω_n on the angle φ (i.e. on the direction of SW propagation in the film plane). This effect takes place only if the following conditions are fulfilled:

$$N_{xx}^a \neq N_{yy}^a, N_{xy}^a \neq 0 \tag{20}$$

i.e. if anisotropy changes ellipticity of polarization of the propagating SW in the plane perpendicular to the direction of equilibrium magnetization $\boldsymbol{M}_0$. This conclusion is in accordance with the analogous conclusion for the infinite media (see e.g. Gurevich 1973).

For the particular cases of crystallographic orientations (111), (100), (110) demagnetization factors of anisotropy have the following values (Belyakov 1988):

(i) orientation (111)

$$N_{xx}^a = N_{yy}^a = -2H^A/M_0 \qquad N_{zz}^a = -2H^A/3M_0, \qquad N_{xy}^a = 0 \tag{21}$$

(ii) orientation (100)

$$N_{xx}^a = N_{yy}^a = N_{xy}^a = 0 \qquad N_{zz}^a = -2H^A/M_0 \tag{22}$$

(iii) orientation (110)

$$N_{xx}^a = -3H^A(1-\cos 2\Psi)/2M_0 \qquad N_{yy}^a = -3H^A(1+\cos 2\Psi)/2M_0$$
$$N_{zz}^a = -H^A/M_0 \qquad N_{xy}^a = -3H^A \sin 2\Psi/2M_0 \tag{23}$$

where Ψ is the angle between the axis x and the crystallographic axis $\langle 100\rangle$ in the film plane.

As is clear from (20) and (21)–(23), the angular dependence of the SW eigenfrequency exists only in the case of the orientation (110). In films with the orientations (111) and (100) the influence of the cubic anisotropy on the SW spectrum is equivalent to the change in magnitude of the internal magnetic field $H_i \rightarrow \tilde{H}_i$, and leads to the frequency shift of the SW dispersion curves. The SW dispersion equation for these orientations has the form of the SW dispersion equation in isotropic FF (see (52) in Kalinikos and Slavin 1986):

$$\omega_n^2 = (\tilde{\omega}_H + \alpha\omega_M k_n^2)(\tilde{\omega}_H + \alpha\omega_M k_n^2 + \omega_M P_{nn}) \tag{24}$$

where $\tilde{\omega}_H = \gamma\tilde{H}_i$ and $\tilde{H}_i$ has the following values:

(i) orientation (111)

$$\tilde{H}_i = H_{0z} - M_0 - 4H^A/3$$

(ii) orientation (100)

$$\tilde{H}_i = H_{0z} - M_0 + 2H^A.$$

The influence of uniaxial perpendicular anisotropy on the SW spectrum in the perpendicularly magnetized FF is also equivalent to the change in the internal magnetic field for the value $2H^u$. So the SW spectrum in this case is also described by equation (24) where $\tilde{H}_i = H_{0z} - M_0 + 2H^u$.

It is interesting to note that although the SW eigenfrequency (24) is always affected by crystallographic anisotropy the SW group velocity

$$V_{gn} = \partial\omega_n/\partial k_\xi = (\omega_M\tilde{\omega}_H/2\omega_n)(\partial P_{nn}/\partial k_\xi) \tag{25}$$

is in some cases practically independent of the influence of anisotropy. For example, in the FF with unpinned surface spins in the longwave part of the SW spectrum, the expression for the group velocity of the lowest SW mode ($n = 0$) has the form

$$V_{g0} = \omega_M L/4 \tag{26}$$

i.e. it is determined only by the film thickness and the value of equilibrium magnetization M_0.

7. Tangential magnetization

We obtain the explicit expression for the SW dispersion equation in tangentially magnetized FF ($\theta = 90°$) from (14) and (12) using the expressions for the angle functions A, C, D, E given in Kalinikos and Slavin (1986):

$$(T^{yx}_{nn}\omega_n - \omega_M 2\sin\varphi Q^{yx}_{nn})^2 = (\Omega^x_n + \omega_M - \omega_M P^{xx}_{nn} + \omega_M N^a_{xx}) \times (\Omega^y_n + \omega_M \sin^2\varphi P^{yy}_{nn} + \omega_M N^a_{yy}) - \omega^2_M (T^{yx}_{nn} N^a_{xy})^2. \tag{27}$$

Let us consider some particular cases of FF crystallographic orientation. In the case of a tangentially magnetized cubic FF demagnetization factors of anisotropy have the following values (Belyakov 1988):

(i) orientation (111)

$$N^a_{xx} = -2H^A/M_0 \qquad N^a_{yy} = N^a_{zz} = -H^A/M_0 \qquad N^a_{xy} = \sqrt{2}H^A \sin 3\varphi_M/M_0$$

where φ_M is the angle between the M_0 and the crystallographic axis $\langle 100\rangle$ in the film plane

(ii) orientation (100)

$$N^a_{xx} = N^a_{xy} = 0 \qquad N^a_{zz} = -H^A(\cos 4\varphi_M + 3)/2M_0$$
$$N^a_{yy} = 3H^A(\cos 4\varphi_M - 1)/2M_0 \tag{28}$$

(iii) orientation (110)

$$N^a_{xx} = -3H^A \sin^2\varphi_M/M_0 \qquad N^a_{yy} = -9H^A \sin^2 2\varphi_M/4M_0$$
$$N^a_{zz} = -H^A(\sin^4\varphi_M + 2\cos^4\varphi_M)/M_0 \qquad N^a_{xy} = 0$$

where φ_M is the angle between the $\boldsymbol{M}_0$ and the crystallographic axis ⟨100⟩ in the film plane.

In the case of perpendicular uniaxial anisotropy in tangentially magnetized film the demagnetization factors are:

$$N^a_{xx} = -2H^u/M_0 \qquad N^a_{yy} = N^a_{zz} = N^a_{xy} = 0. \tag{29}$$

Using the expressions (27)–(29) we may obtain the simple explicit SW dispersion equation for FF of the orientation (111) having cubic and uniaxial anisotropy and unpinned surface spins

$$\omega_n^2 = [\Omega_n + \omega_M - \omega_M P_{nn} - 2\gamma(H^A + H^u)](\Omega_n + \omega_M P_{nn} \sin^2\varphi - \gamma H^A) - 2(\gamma H^A \sin 3\varphi_M)^2 \tag{30}$$

where Ω_n is determined by the formula (12) and $H_i = H_{0z} + H^A$.

Equation (30) can be used for determining the magnetic parameters (equilibrium magnetization and effective magnetic fields of anisotropy) of the FF under consideration by means of measuring the frequency of SW propagating in the FF. In particular, the angle dependence of the SW frequency $\omega_n = f(\varphi_M)$ enables us to determine the effective magnetic field of the cubic anisotropy H^A. The measurement of the SW frequencies corresponding to the different values of the SW wavenumber enables us to distinguish and determine M_0 and the effective magnetic field of the uniaxial anisotropy H^u.

8. Hybridization of the SW dispersion branches

The hybridization (or interaction) of 'unperturbed' diagonal SW modes with different numbers takes place near the spectral points of frequency degeneracy of these modes. The unperturbed dispersion equations for these SW modes are given by the expressions (14), (15), (24), (27), so the condition of frequency degeneracy has the form

$$\omega_n(k_\zeta) = \omega_{n'}(k_\zeta) \qquad n \neq n'.$$

Hybridization of the crossing dispersion branches leads to the formation of dipole 'gaps' in the dipole-exchange SW spectrum of the FF. The decrease of the SW group velocity in the spectral regions of hybridization causes an increase in the spatial attenuation of propagating SW in these regions. In experiments this effect manifests itself in the form of oscillations in the propagation loss characteristic of the experimental device (delay line) (see e.g. Adam *et al* 1978, Kalinkos *et al* 1983, Andreev *et al* 1984) and can be observed in perpendicularly as well as tangentially magnetized FF. We shall consider both these particular cases below.

Using equations (18*a*, *b*) we obtain the explicit dispersion equation describing the effect of hybridization of the SW modes with numbers n and n' in the particular case of perpendicularly magnetized ($\theta = 0$, $\varphi = 0$) anisotropic FF with either totally pinned or totally unpinned surface spins:

$$(\omega_n^2 - \omega^2)(\omega_{n'}^2 - \omega^2) = \omega_M^2 P_{nn'} P_{n'n}(\Omega_n + \omega_M N^a_{yy})(\Omega_{n'} + \omega_M N^a_{yy}) \tag{31}$$

where ω_n, $\omega_{n'}$ are the frequencies of the 'unperturbed' SW determined by equations (15) and (19), and P_{nn} is the matrix element of dipole–dipole interaction determined in Kalinikos and Slavin 1986.

The intensity of hybridization can be evaluated by means of the width of the dipole 'gap' $\delta\omega_{nn'}$ in the SW spectrum of the FF

$$\delta\omega_{nn'} = \omega_M P_{nn'}[(\Omega_n + \omega_M N^a_{yy})(\Omega_{n'} + \omega_M N^a_{yy})]^{1/2}/\omega_0 \tag{32}$$

where $\omega_0 = \omega_n(k_{\zeta 0}) = \omega_{n'}(k_{\zeta 0})$ is the central frequency of the dipole 'gap'.

In the case of transverse ($\varphi = 90°$) SW propagating in tangentially magnetized ($\theta = 90°$) anisotropic FF with either totally pinned or totally unpinned surface spins, the secular dispersion equation (18) yields

$$\begin{aligned}(\omega_n^2 - \omega^2)(\omega_{n'}^2 - \omega^2) &= 2\omega_M^2(a_n a_{n'} - \omega^2)(P^2_{nn'} + 4Q^2_{nn'}) + \omega_M^2(P^2_{nn'} - 4Q^2_{nn'}) \\ &\times [(b^+ - \omega_M P_{nn})(b^+ - \omega_M P_{n'n'}) + (b^- - \omega_M P_{nn})(b^- - \omega_M P_{n'n'})] \\ &- \omega_M^4(P^2_{nn'} - 4Q^2_{nn'})^2 \end{aligned} \tag{33}$$

where

$$a_n = \Omega_n + \omega_M(1 + N^a_{xx} + N^a_{yy})/2$$

$$b^{\pm} = \omega_M(1 + N^a_{xx} - N^a_{yy} \pm i2N^a_{xy})/2$$

ω_n and $\omega_{n'}$ are the frequencies of the 'unperturbed' SW determined by equation (27).

Using (33) it is a straightforward task to derive the expressions for the width of the dipole 'gap' in the spectrum of transverse SW in the tangentially magnetized anisotropic FF:

(i) for the case when SW modes have a similar type of symmetry

$$\delta\omega_{nn'} = \omega_M^2 P_{nn'}[1 + (N^a_{xx} - N^a_{yy})(1 + N^a_{xx} - N^a_{yy})]^{1/2}/\omega_0 \tag{34}$$

(ii) for the case when SW modes have different types of symmetry

$$\delta\omega_{nn'} = \omega_M^2 Q_{nn'}[4(N^a_{xy})^2 + (1 + N^a_{xx} - N^a_{yy})(P_{nn} + P_{n'n'})]^{1/2}/\omega_0 \tag{35}$$

where ω_0 is the central frequency of the dipole 'gap'.

Strong dependence of the amplitude of oscillations of the experimental transmission-loss characteristic on the relative orientation of the external bias magnetic field $\boldsymbol{H}_0$ and the crystallographic axis in the film plane was observed in the experiments of Andreev *et al* (1984) for the transverse surface SW in a tangentially magnetized YIG film of orientation (110).

The equations (34), (35), (28) enable us to analyse the influence of crystallographic anisotropy on the width of dipole 'gaps' in the SW spectrum of FF of the orientation (110). Using (34), (35) we calculated the angle dependencies of the 'gap' width $\delta\omega_{nn'} = f(\varphi_M)$ in the YIG film with unpinned surface spins for the case of hybridization of the lowest SW modes. The angle dependence of the width of the 'gap' caused by hybridization of the lowest SW ($n = 0$) with odd and even numbered SW modes is presented in figure 7. It can be seen that the angular dependence is determined by the symmetry of the interacting SW modes. In the case of interaction of SW modes with a similar type of symmetry (when n and n' are either both even or both odd), the maximum width of the dipole 'gap' in the spectrum is obtained when the direction of $\boldsymbol{H}_0$ coincides with the direction of the axis ⟨110⟩.

The minimum width is obtained when the $\boldsymbol{H}_0$ is directed along the axis ⟨111⟩, and when $\boldsymbol{H}_0$ is directed along the axis ⟨100⟩ the 'gap' width has a local maximum. The values of both maxima are quite close, so the $\delta\omega_{\langle 110\rangle}$ is only several per cent greater than $\delta\omega_{\langle 100\rangle}$.

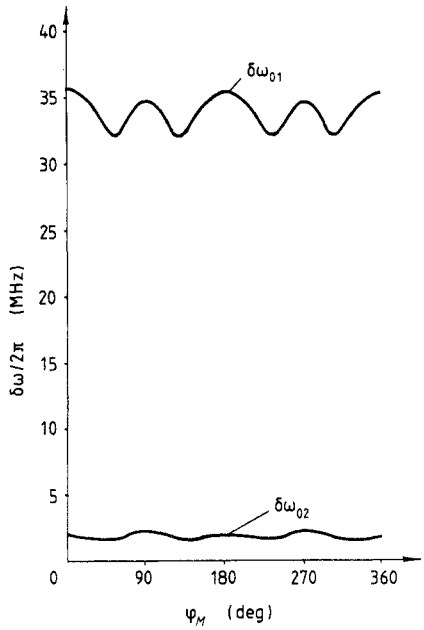

Figure 7. Angular dependence of the width of dipole 'gap' in the dipole-exchange SW spectrum of the YIG film having cubic anisotropy and unpinned surface spins. $H_0 = 1000$ Oe; $4\pi M_0 = 1750$ Oe; $H^A = -42$ Oe; $K_\zeta L = 0.1$.

In the case of interaction of SW modes with different types of symmetry the maximum 'gap' width is obtained when the $\boldsymbol{H}_0$ is directed along the axis ⟨111⟩; the minimum corresponds to the direction ⟨100⟩, and when $\boldsymbol{H}_0$ is directed along the axis ⟨110⟩ the value of $\delta\omega_{\langle 110\rangle}$ is quite close to that of $\delta\omega_{\langle 100\rangle}$.

The calculated angle dependencies of the dipole 'gap' width in the SW spectrum are in good qualitative agreement with the results of the experiments by Andreev *et al* (1984) where the width of the dipole 'gap' in the SW spectrum in the YIG film was measured while the direction of the external magnetic field lying in the film plane was changed.

The expressions obtained, (34) and (35), for the dipole 'gap' width in the SW spectrum of FF enable us to determine the directions of magnetization corresponding to the minimum values of the 'gap' widths in the cases of arbitrary crystallographic orientations of the FF. The results of such theoretical estimations might appear to be useful in development of SW signal processing devices as by choosing the right direction of the external magnetic field it is possible to diminish significantly the oscillations of the transmission loss characteristic of the SW device under consideration.

9. Conclusions

In this paper we obtained the exact expressions for the dipole-exchange spin wave spectrum in anisotropic ferromagnetic films where all the significant interactions were taken into account.

We also obtained simple approximate dispersion equations which enable us to make qualitative analysis of the influence of the crystallographic anisotropy on the SW spectrum of the FF both in the dipole-exchange and the non-exchange limits.

The results of this analysis and the results of numerical calculations using the exact SW dispersion equations (see sections 3, 4) appear to be in good agreement with the

results of the experiments performed in YIG and Mn–ferrospinel films (see Andreev *et al* 1984, Anfinogenov *et al* 1986, Anfinogenov *et al* 1988) as well as with the results of the existing non-exchange theory of SW spectrum of anisotropic FF (see e.g. Beregov 1984, Gieniusz and Smoczinski 1987).

The consideration of the influence of crystallographic anisotropy on the dipole-exchange SW spectra of FF presented in this paper may turn out to be vital for the development of SW signal processing devices using monocrystalline hexaferrite films as the relative values of effective magnetic fields of anisotropy in such films are much greater than in YIG films.

The general theory of SW spectra in ferromagnetic films developed in this paper can also be used as a basis for development of the theory of nonlinear SW effects and the theory of SW relaxation in anisotropic monocrystalline FF.

Appendix

This appendix gives the determination of the equilibrium orientation of the saturation magnetization $\boldsymbol{M}_0$ in the aniosotropic film.

The angles φ and θ, characterizing the equilibrium orientation of the saturation magnetization $\boldsymbol{M}_0$, can be determined from the conditions of minimization of the density of magnetic energy of the ferromagnetic sample (see e.g. Gurevich 1973, Lax and Batton 1962). These conditions for the ferromagnetic ellipsoid having cubic and uniaxial anisotropy are given in Belyakov (1988) in the form

$$\begin{aligned} -(\partial E_H/\partial\theta) &= (\partial E_m/\partial\theta) + (\partial E_A/\partial\theta) + (\partial E_u/\partial\theta) \\ -(\partial E_H/\partial\varphi) &= (\partial E_m/\partial\varphi) + (\partial E_A/\partial\varphi) + (\partial E_u/\partial\varphi). \end{aligned} \tag{A1}$$

where E_H is the density of energy of Zeeman interaction with bias external magnetic field, E_m is the density of demagnetization energy, while E_A and E_u are the densities of energy of cubic and uniaxial anisotropy correspondingly. The expressions for the densities of all the types of magnetic energy mentioned above are given in Belyakov (1988) for the general case of arbitrary orientation of the axis of the ferromagnetic ellipsoid with respect to the orientations of the axis of the cubic crystal and the axis of uniaxial anisotropy. Since we are dealing with ferromagnetic films we shall write here the system (A1) for the particular case of a thin ferromagnetic disc of crystallographic orientation (111) having the axis of uniaxial anisotropy perpendicular to its plane (see Belyakov 1988):

$$\begin{aligned} H_0 W &= \tfrac{1}{2}(2H^u - M_0)\sin 2\theta + H^A P_\theta \\ H_0 \sin\theta \sin\theta_H \sin(\varphi_{H_0} - \varphi_M) &= H^A P_\varphi \end{aligned} \tag{A2}$$

where

$$W = \cos\theta \sin\theta_H \cos(\varphi_{H_0} - \varphi_M) - \sin\theta\cos\theta_H \tag{A3}$$

$$P_\theta = \tfrac{1}{3}\sin\theta[\cos\theta(3 - 7\cos 2\theta) + \sqrt{2}\sin 3\theta\cos 3\varphi_M] \tag{A4}$$

$$P_\varphi = -\sqrt{2}\sin^3\theta\cos\theta\sin 3\varphi_M \tag{A5}$$

$$H^u = K^u/M_0 \qquad H^A = K_1/M_0 \tag{A6}$$

and K^u and K_1 are the constants of uniaxial and cubic anisotropy. The angles φ_{H_0} and

φ_M are determined with respect to the crystallographic axis ⟨112⟩ in the film plane (see figure 1).

If the angles φ_H, φ_{H_0} and θ_H determining the orientation of the external bias magnetic field $\boldsymbol{H}_0$ are given, the angles φ and θ characterizing the equilibrium orientation of the saturation magnetization $\boldsymbol{M}_0$ in the particular case specified above can be obtained from the numerical solution of the transcendental system (A2) taking into account that (see figure 1)

$$\varphi = \varphi_{H_0} + \varphi_H - \varphi_M.$$

In the case of different crystallographic orientations of FF the general system of equations (A1) must be solved to determine the orientation of the saturation magnetization $\boldsymbol{M}_0$.

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